Locating-dominating sets in twin-free graphs

Florent Foucaud
Université Blaise Pascal, Clermont-Ferrand, France

joint work with:

Michael A. Henning (University of Johannesburg, South Africa)

Christian Löwenstein (Universität Ulm, Germany)

Thomas Sasse (Universität Ulm, Germany)

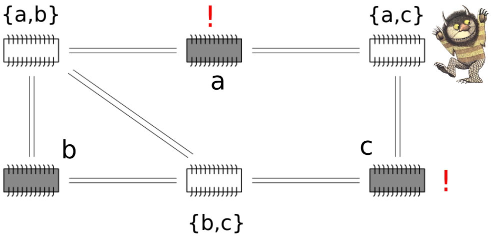
June 2015

Definition - Locating-dominating set (Slater, 1980's)

$D \subseteq V(G)$ locating-dominating set of G:

- for every $u \in V$, $N[v] \cap D \neq \emptyset$ (domination).
- $\forall u \neq v$ of $V(G) \setminus D$, $N(u) \cap D \neq N(v) \cap D$ (location).

Motivation: fault-detection in networks.
$\rightarrow$ The set D of grey processors is a set of fault-detectors.



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Notation. location-domination number $LD(G)$: smallest size of a locating-dominating set of G

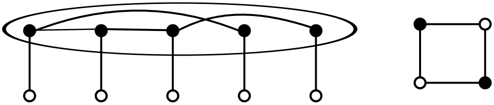
Domination number: $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$

Location-domination number: $LD(P_n) = \left\lceil \frac{2n}{5} \right\rceil$

Theorem (Domination bound, Ore, 1960's)

G graph of order n, no isolated vertices. Then $\gamma(G) \leq \frac{n}{2}$.

Tight examples:



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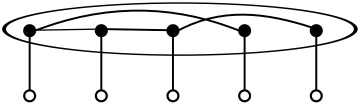
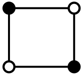
Tight examples:  

Theorem (Location-domination bound, Slater, 1980's)

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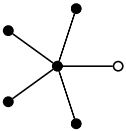
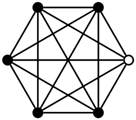
Tight examples:  

Upper bounds

Theorem (Domination bound, Ore, 1960's)

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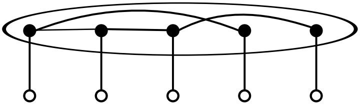

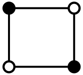

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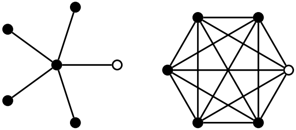

Remark: tight examples contain many twin-vertices!!

Upper bound: a conjecture

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Remark:

- twins are easy to detect
- twins have a trivial behaviour w.r.t. location-domination

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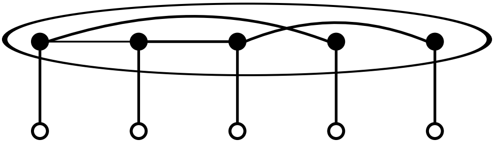
If true, tight: 1. domination-extremal graphs



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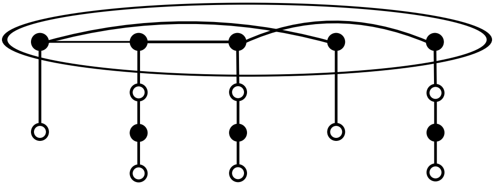
If true, tight: 2. a similar construction



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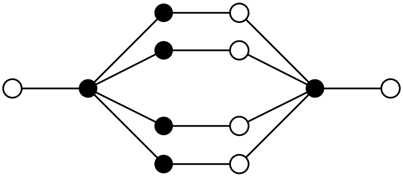
If true, tight: 3. a family with domination number 2



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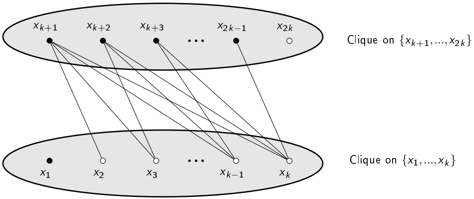
If true, tight: 4. a *dense* family with domination number 2



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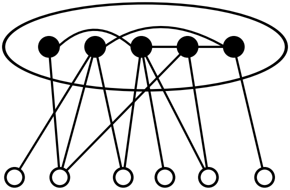
Theorem (Garijo, González & Márquez, 2014)

Conjecture true if G has independence number $\geq n/2$.
(in particular, if bipartite)

Proof: every vertex cover is a locating-dominating set



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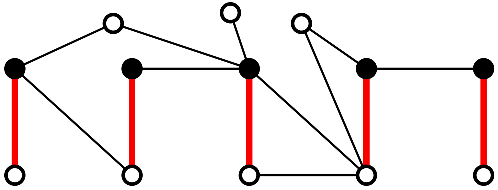
$\alpha'(G)$: matching number of G

Theorem (Garijo, González & Márquez, 2014)

If G has no 4-cycles, then $LD(G) \leq \alpha'(G) \leq \frac{n}{2}$.

Proof:

- Consider special maximum matching M
- Select one vertex in each edge of M



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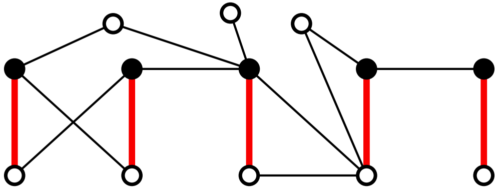

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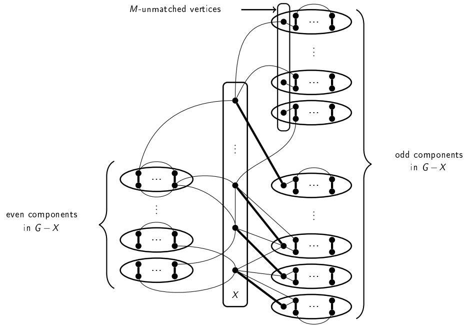
Theorem (F., Henning, 2015+)

Conjecture true if G is cubic.

Proof: Involved argument using maximum matching and Tutte-Berge theorem.



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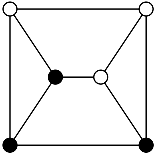
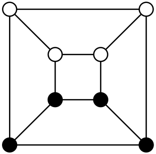
Bound is tight:





Question

Do we have $LD(G) = \frac{n}{2}$ for other cubic graphs?

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Question

Are there twin-free (cubic) graphs with $LD(G) > \alpha'(G)$?

(if not, conjecture is true)

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Theorem (F., Henning, Löwenstein, Sasse, 2014+)

Conjecture true if G is split graph or complement of bipartite graph.

Line graph of G: intersection graph of the edges of G.

Theorem (F., Henning, 2015+)

Conjecture true if G is line graph.

Proof: By induction on the order, using edge-locating-dominating sets

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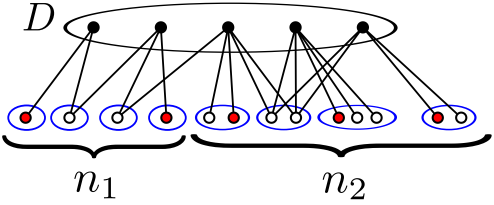
Proof: • There exists a dominating set D such that each vertex has a private neighbour. We have $|D| \leq n_1 + n_2$.



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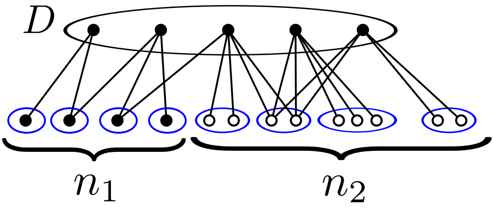

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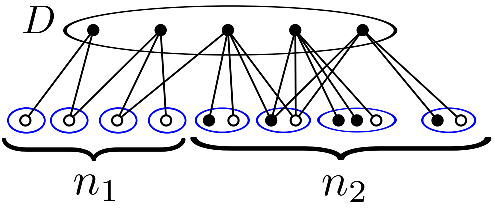

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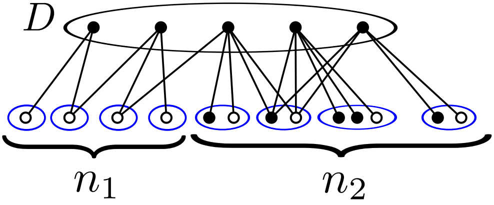
• $\min\{|D| + n_1, n - n_1 - n_2\} \leq \frac{2}{3}n$

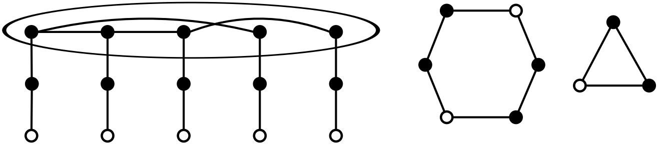


total dominating set: each vertex has a neighbour in the dominating set.

Theorem (Total domination bound, Cockayne, Dawes, Hedetniemi, 1980)

G graph of order n, no isolated vertices/edges. Then $\gamma_t(G) \leq \frac{2}{3}n$.

Tight examples for γ_t:



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$LTD(G)$: size of smallest locating-total dominating set

Conjecture (F., Henning, 2014)

G graph of order n, no isolated vertices/edges, twins. Then $LTD(G) \leq \frac{2}{3}n$.

Upper bound: a conjecture - general bound

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Theorem (F., Henning, 2015+)

Conjecture true for graphs with no 4-cycles and for line graphs.

Open problems

Conjecture (Garijo, González & Márquez, 2014)

G graph of order n, no isolated vertices, no twins. Then $LD(G) \leq \frac{n}{2}$.

- Which bipartite graphs satisfy $LD(G) = \frac{n}{2}$? (known for trees)
- Are there twin-free graphs with $LD(G) > \alpha'(G)$?

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THANKS FOR YOUR ATTENTION